Formalizing (Non)iterativity and the Computation of Rule Application Modes

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April 28, 2022

Iterative and noniterative

- The terms *iterative* vs. *noniterative* have their origin in rule-based phonology, but even in that context what they mean depends on the theory.
- Outside of rule-based phonology, they serve as descriptive labels for certain phonological maps, but it is unclear to what extent they reflect actual properties of those maps.
- Viewing phonological maps in terms of their computational properties leads to a more formal and precise understanding of what it means for a map to be iterative, or noniterative, or both, or neither...

Simultaneous application

'To apply a rule, the entire string is first scanned for segments that satisfy the environmental constraints of the rule. After all such segments have been identified in the string, the changes required by the rule are applied simultaneously' (Chomsky and Halle, 1968).

- Assumption of noniterativity = rules cannot reapply to their own outputs.

(1) [−cons] → [+nasal] / [+nasal] __

a. NVVV → NṼVV

Simulating iteration

(2) $[-\text{cons}] \rightarrow [+\text{nasal}] \ / \ [+\text{nasal}] \ ([-\text{cons}])^* \ \_\_$

a. $[-\text{cons}] \rightarrow [+\text{nasal}] \ / \ [+\text{nasal}] \ \_\_$

b. $[-\text{cons}] \rightarrow [+\text{nasal}] \ / \ [+\text{nasal}] \ [-\text{cons}] \ \_\_$

c. $[-\text{cons}] \rightarrow [+\text{nasal}] \ / \ [+\text{nasal}] \ [-\text{cons}][-\text{cons}] \ \_\_$

d. $\text{NVVV} \rightarrow \text{N}\tilde{\text{V}}\tilde{\text{V}}\tilde{\text{V}}$

Directional rules

- Criticisms of parenthesis-star motivated proposals in which rules *can* apply to their own outputs (Johnson, 1972; Howard, 1972; Lightner, 1972; Anderson, 1974; Kenstowicz and Kisseberth, 1977).
- In some theories, direction is stipulated:

$$[-\text{cons}] \rightarrow [+\text{nasal}] \;/\; [+\text{nasal}] \;\_\_$$

left-to-right	right-to-left
NVVV → NṼṼṼ	NVVV → NṼVV

Directional rules

- In other theories, direction is determined by the rule's form:

(3) a. [−cons] → [+nasal] / [+nasal] __ (left-to-right)
    b. [−cons] → [+nasal] / __ [+nasal] (right-to-left)

- Noniteration remains available as an ad hoc solution when needed.
- The availability of both options is more official in theories with an iterativity parameter (e.g., Archangeli and Pulleyblank, 1994).

Enter OT (Prince and Smolensky, 1993)

- Optimality Theory is inherently iterative, in that iteration is optimal.

/NVVV/	*[+nas][−nas]	IDENT-NAS
[NVVV]	*!	
[NṼVV]	*!	*
[NṼṼV]	*!	**
☞[NṼṼṼ]		***

Feature not a bug

- Emergent Noniterativity Hypothesis (Kaplan, 2008): No formal entity in phonological grammars may require noniterativity.
- Noniterativity is always epiphenomenal and can be explained by other means.

Emergent noniterativity

(4) Lango (Nilo-Saharan; Noonan, 1992)

a. /bɔ̀ŋɔ́-ní/ → [bɔ̀ŋó-ní], ‘your (sg.) dress’

b. /cɔ̀ŋɔ̀-ní/ → [cɔ̀ŋò-ní], ‘your (sg.) beer’

- Positional licensing: suffix [ATR] needs to be linked to the root (Kaplan, 2008).

‘True noniterativity’

- But, some cases of noniterativity cannot be reanalyzed as emergent in this way (Ampofo and Rasin 2021,McCollum under review).

Defining (non)iterativity

- Extension: NVVV → NṼVV
- Intensional description:

  [−cons] → [+nasal] / [+nasal] __ (simultaneous application/−iterative)

  [−cons] → [+nasal] / [+nasal] __ (right-to-left iterative)

  [−cons] → [+nasal] / [+nasal, +cons] __ (left-to-right, right-to-left, −iterative/simultaneous)

Formal Language Theory

- FLT models of phonological maps reflect more invariant properties compared to their intensional descriptions.
- Finite-state models typically implement iteration vs. noniteration by matching rule contexts on the output vs. input string, respectively (Kaplan and Kay, 1994; Hulden, 2009; Gorman and Sproat, 2021).

Matching context on input

N	V	V	V
N	Ṽ		

N	V	V	V
N	Ṽ	V	

N	V	V	V
N	Ṽ	V	V

Matching context on output

N	V	V	V
N	Ṽ		

N	V	V	V
N	Ṽ	Ṽ	

N	V	V	V
N	Ṽ	Ṽ	Ṽ

Input/output and +/- iterative

- It's tempting to then define iterative as 'output-based' and noniterative as 'input-based', in terms of the computations they require.
- But, the sets of maps that can be generated with these computation types are not disjoint, and the distinction is often irrelevant.
- GOAL: isolate those cases for which the distinction does matter, i.e., those cases for which input-based computation is *necessary* to prevent otherwise inescapable iteration.

Significance

1. Evidence that a computational(ly-informed) theory of phonology needs access to both input and output structure.
2. More thorough understanding of when each type of computation is needed and why.
3. More precise definitions of *iterative* and *noniterative* that aren't tied to a particular theory's intensional descriptions.

Rules as functions

- Zooming in for now on rules of this form:

(5)
a. A → B / C __
b. A → B / __ C
c. where A and C are finite sets of strings

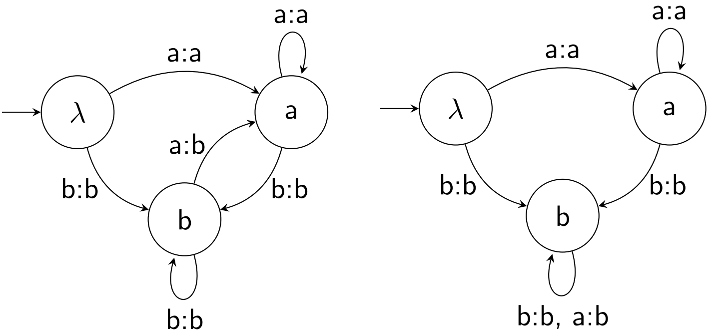
- In other words, 'local processes'
- Doing so enables the use of more restrictive finite-state models that make the need for input/output structure more clear.
- Will return to rules with two-sided contexts later.

ISL and OSL

a → b / b __

Input Strictly Local

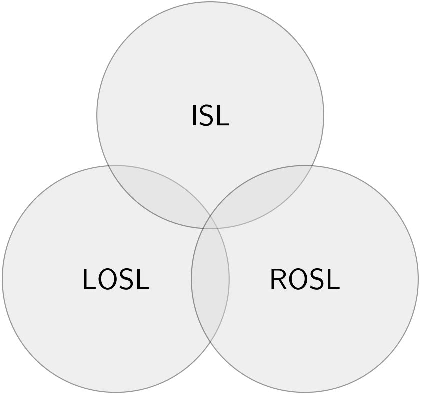
(Left) Output Strictly Local



baaa → bbaa

baaa → bbbb

Relationship among classes



Chandlee (2014); Chandlee et al. (2015)

Substitution (non-dissimilatory)

$$[-\text{cons}] \rightarrow [+\text{nasal}] \ / \ [+\text{nasal}] \ \_\_$$

+iterative	−iterative
NVVV → NṼṼṼ	NVVV → NṼVV
LOSL?	ISL?

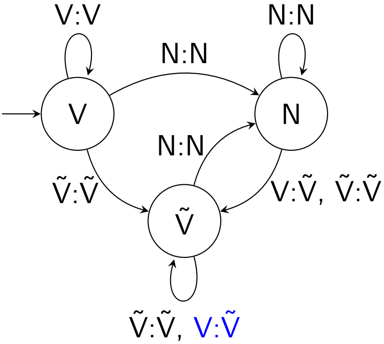
Substitution (non-dissimilatory)

$$[-\text{cons}] \rightarrow [+\text{nasal}] \ / \ [+\text{nasal}] \ \_\_$$

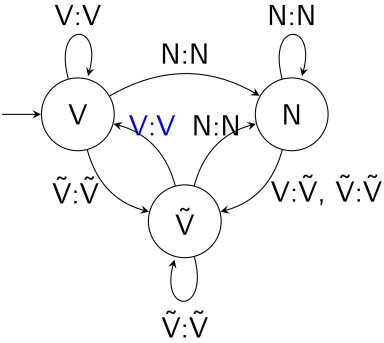
+iterative	−iterative
$\text{NVVV} \rightarrow \text{N}\tilde{\text{V}}\tilde{\text{V}}\tilde{\text{V}}$	$\text{NVVV} \rightarrow \text{N}\tilde{\text{V}}\text{VV}$
LOSL	ISL ∩ LOSL

OSL and noniterativity

OSL (+iterative)



OSL (−iterative)



Triggers and Outputs

Let $\textsc{Triggers}$ be the set of segments that trigger a process, and let $\textsc{Outputs}$ be the set of segments that result from that process.

A phonological map is

1. *output-nondistinct* if $\textsc{Outputs} \cap \textsc{Triggers} \neq \varnothing$.
2. *output-distinct* if $\textsc{Outputs} \cap \textsc{Triggers} = \varnothing$.

Triggers and Outputs

NVVV → NṼṼṼ	NVVV → NṼVV
TRIGGERS = {N, Ṽ}	TRIGGERS = {N}
OUTPUTS = {Ṽ}	OUTPUTS = {Ṽ}
output-nondistinct	output-distinct

Bengali ATR harmony

{ɛ, ɔ} → {e, o} before a high vowel (Mahanta, 2007).

kɔt^{h}a	‘spoken words’	kɔlpo	‘resembling’
kot^{h}ito	‘uttered’	kolpito	‘invented’
kɔthoniyo	‘speakable’	kɔlponiyo	‘imaginable’

/ɔɔi/ → [ɔoi]

Bengali ATR harmony

- TRIGGERS = {i, u} (only high vowels can trigger harmony)
- OUTPUTS = {e, o}
- Output-distinct
- ISL $\cap$ ROSL

Central Crimean Tatar labial harmony

Initial [+rd] vowels trigger labial harmony on the next high vowel (McCollum and Kavitskaya, 2018).

/tuz-lɯɣ-ɯ/	[tuz-luɣ-ɯ]	‘salt’
/kyz-lig-i/	[kyz-lyg-i]	‘autumn’
/toz-lɯɣ-ɯ/	[toz-luɣ-ɯ]	‘dust’
/køz-lig-i/	[køz-lyg-i]	‘eye’

All forms marked for NMZR-POSS.3S.

Crimean Tatar labial harmony

- TRIGGERS = {y, ø, u, o}
- OUTPUTS = {y, ø, u, o}

Crimean Tatar labial harmony

- $\textsc{Triggers} = \{^{\#}$y, $^{\#}$ø, $^{\#}$u, $^{\#}$o$\}$ (only initial vowels are triggers)
- $\textsc{Outputs} = \{$y, ø, u, o$\}$
- Output-distinct
- ISL ∩ LOSL

Crimean Tatar labial harmony

$V_{-rd} \rightarrow [+\text{rd}] \ / \ \# \ V_{+rd} \ \_\_$

*#[+rd, −rd] (McCollum and Kavitskaya, 2018)

Akan serial verbs

[+ATR] spreads leftward to the next vowel, but no further (Ampofo and Rasin, 2021).

/tɔ fa di/	[tɔ fæ di]	'buy, take, and eat'
/tɔ fa bɔ di/	[tɔ fa bo di]	'buy, take, crack, and eat'
/tɔ di su/	[to di su]	'buy, eat, and cry'

Akan serial verbs

- $\textsc{Triggers} = \textsc{Outputs} = \{$i, e, æ, o, u$\}$
- Output-nondistinct
- ISL only

Substitution (non-dissimilatory)

ISL	ISL ∩ OSL	OSL
output-nondistinct single target	output-distinct	output-nondistinct multiple targets

Dissimilation

(6) L → R / L __

- TRIGGERS = {L}
- OUTPUTS = {R}
- Output-distinct
- Predicts both ISL and OSL will produce a noniterative map.

Dissimilation

(7) L → R / L __

- ISL (and simultaneous application): LLLL → LRRR
- LOSL (and iterative application): LLLL → LRLR

Tianjin tone sandhi (Chen, 1986, 2000)

R → H / __ R	RRR → HHR
F → L / __ F	FFF → FLF
L → R / __ L	LLL → LRL

Rhythmic law

(8) $V{:} \rightarrow V \;/\; V{:}\ C_0$ __

Slovak (Indo-European; Slovakia)

/čiːt-aːm/	[čiːt-am]	'read-1S'
/čiːt-aːv-aːm/	[čiːt-av-am]	(frequentive)

Githabal (Australian)

/nuːn-daːŋ/	[nuːn-daŋ]	'too hot'
/djalum-baː-daːŋ-beː/	[djalum-baː-daŋ-beː]	'is certainly right on the fish'

(Kenstowicz and Kisseberth, 1979)

Dissimilation (summary)

ISL	ISL ∩ OSL	OSL
identical span		alternating pattern

- Output-distinctness is irrelevant; these rules are self-*bleeding*.
- What is relevant is whether the rule generates an alternating pattern or a span of identical segments.
- ‘Forced choice’ assumption of (non)iterativity, though we often won’t have the data to tell.

‘Anchored’ deletion

(9) $V \rightarrow \varnothing$ / __ #

- ISL (and simultaneous application): CVVV → CVV
- ROSL (and iterative application): CVVV → C

'Overlap' deletion

(10) $V \rightarrow \varnothing$ / __ V

ISL (simultaneous)	aeiou → u
LOSL (left-to-right iter)	aeiou → eiou → iou → ou → u
ROSL (right-to-left iter)	aeiou → aeiu → aeu → au → u

Deletion (summary)

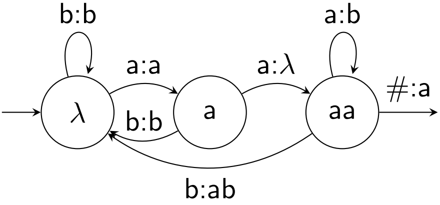
ISL	ISL ∩ OSL	OSL
anchored single target	overlap	anchored multiple targets

Interim summary

	ISL	ISL ∩ OSL	OSL
substitution	output-nondist single target	output-distinct	output-nondist multiple targets
dissimilation	identical span		alternating pattern
deletion	anchored single target	overlap	anchored multiple targets

Rules with two-sided contexts

(11) a → b / a __ a



aaaa → abba

Rules with two-sided contexts

- These rules are necessarily ISL, but do they ever iterate?
- Survey of PBase (Mielke, 2008):

	Potentially iterative	Total
Substitution	8	672
Deletion	37	207
Epenthesis	—	287

(12) Iterative epenthesis

a. $\varnothing \rightarrow$ ab / a __ b

Potentially iterative, but...

(13) Mundari (Austro-Asiatic; India; Cook, 1965, pg. 61)

a. {u, o} → [w] / V __ V

b. /kiŭa/ [kiwa] 'chin'

c. /heŏa/ [hewa] 'accustom'

- Need to see what happens to /euua/

Rhythmic syncope

(14) $V \rightarrow \varnothing$ / VC __ CV

Maithili (Indo-European; India; Yadav, 1996, pg. 51)

/nikəl-ət-ah/	[nikəl_tah] ~ [nik_lətah], *[nik_l_tah]	‘will come out’
/ləṭək-əl-əuk/	[ləṭ_kələuk] ~ [ləṭək_ləuk], *[ləṭ_k_ləuk]	‘(your) hung’

Rhythmic syncope

- Definitely iterates, but is it two-sided?
- (CV)^n, if n is odd...

	C	V	C	V	C	V	C	V	C	V
$V \rightarrow \varnothing$ / VC ___ CV				$\varnothing$		_		$\varnothing$		
$V \rightarrow \varnothing$ / VC ___				$\varnothing$		_		$\varnothing$		_

Rhythmic syncope

- Definitely iterates, but is it two-sided?
- $(CV)^n$, if n is even...

	C	V	C	V	C	V	C	V
$V \rightarrow \varnothing$ / VC __ CV				$\varnothing$		_		
$V \rightarrow \varnothing$ / VC __				$\varnothing$		_		$\varnothing$

Two-sided iterative rules

- Neither ISL nor OSL, because they need both.
- See Hao and Bowers (2019); Bowers and Hao (2020) for one solution: tier-based synchronized strictly *k*-local (*k*-TSSL) functions

Conclusions

- In rule-based phonology, *iterative* = a rule that reapplies to its own output. If all rules apply this way, then all maps are iterative.
- In OT, *noniterative* = epiphenomenal, or maybe 'potentially problematic'.
- Once we step away from rules/constraints, we can reserve the terms *iterative* and *noniterative* for those cases that necessitate output- or input-based computation, respectively.
  - Or, we could just dispense with these terms entirely!

Conclusions

- The results presented in this talk identify when we need each computation type and why.
- The original goal of having only one rule application mode was motivated by a need for restrictiveness.
- Having access to both input and output structure does not necessarily compromise that goal, as subregularity ensures restrictiveness.

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Thank you!

Appendix: Maps that are +/− iterative?

- In vowel harmony with *icy targets* (Jurgec, 2011), a vowel undergoes the harmony but then blocks it.

(15) Karajá (Macro-Jê; Central Brazil; Ribeiro, 2003)

a.	/bɛdɔ-dĩ/	[bedoni]	'a type of *filhote* (fish species)'
b.	/krɔbɪ-dĩ/	[krɔbini]	'a type of monkey'

Appendix: Maps that are +/− iterative?

- Input *tier-based* strictly local function (Burness et al., 2021)
- Alphabet: {C, ɔ, ɛ, ɪ, ʊ, o, e, i, u}
- Tier: {o, e, i, u, ɪ, ʊ}