Recursive Schemes for Phonological Analysis

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Introduction

- Central goal: identify abstract computational universals of phonological patterns.
  - e.g., phonological rules are *regular relations* (Johnson, 1972; Kaplan and Kay, 1981, 1994)
    - i.e., can be computed for any string using a fixed amount of memory
  - More recent work has posited further constraints on this memory
    - e.g., phonological patterns are *subregular* (Heinz, 2009; Heinz and Lai, 2013; Chandlee and Heinz, 2018, a.o.)
- A frequent criticism of this approach is a disconnect with the intensional representations of generalizations that phonologists use and recognize.

Introduction

- We present a computational formalism that retains computational restrictiveness and better aligns with commonly-held assumptions about phonological representations and grammars.
- Specifically, we introduce *Boolean Monadic Recursive Schemes (BMRSs)* and demonstrate their utility for phonological analysis.
- Advantages:
    1. well-understood complexity bound
    2. means of implementing phonological substance

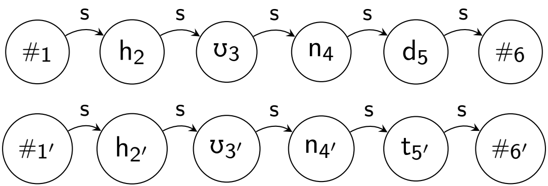
Graph transduction

- We represent strings as graphs and processes as maps from an input to an output graph.

(1) German /hʊnd/ $\mapsto$ [hʊnt], 'dog'

Input:

Output:



Graph transduction

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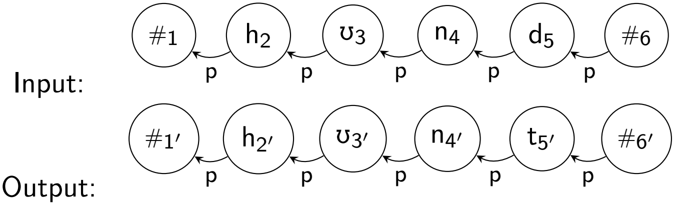
(2) German /hʊnd/ ↦ [hʊnt], ‘dog’



Output predicates

$\text{voice}_o(x) = \ldots$ (under what conditions is an output segment specified as +voice?)

Boolean Monadic Recursive Schemes

$$\text{voice}_o(x) = \texttt{if } X \texttt{ then } B \texttt{ else } Y$$

Output predicates

$$\text{voice}_{o}(x) \quad = \quad \texttt{if}\ \text{sonorant}(x)\ \texttt{then}\ \top\ \texttt{else}\ \texttt{if}\ \#(s(x))\ \texttt{then}\ \bot\ \texttt{else}\ \text{voice}(x)$$

- $\top$ = True
- $\bot$ = False

Output predicates

Input:	#	h	ʊ	n	d	#
	1	2	3	4	5	6
Output:						
	1′	2′	3′	4′	5′	6′

$$\text{voice}_o(x) = \texttt{if } \text{sonorant}(x) \texttt{ then } \top \texttt{ else } \texttt{if } \#(s(x)) \texttt{ then } \bot \texttt{ else } \text{voice}(x)$$

Output predicates

Input:	#	h	ʊ	n	d	#
	1	2	3	4	5	6
Output:			ʊ	n		
	1′	2′	3′	4′	5′	6′

$$\text{voice}_o(x) = \texttt{if } \text{sonorant}(x) \texttt{ then } \top \texttt{ else}$$
$$\texttt{if } \#(s(x)) \texttt{ then } \bot \texttt{ else}$$
$$\text{voice}(x)$$

Output predicates

Input:	#	h	ʊ	n	d	#
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Output predicates

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$$\text{voice}_o(x) \;=\; \texttt{if } \text{sonorant}(x) \texttt{ then } \top \texttt{ else}$$
$$\texttt{if } \#(s(x)) \texttt{ then } \bot \texttt{ else}$$
$$\text{voice}(x)$$

Advantages

- Similar to OT, BMRSs employ a hierarchy of constraint-like predicates that refer to particular structures in the input/output.
- These structures may alternately **license** or **block** particular feature values from surfacing.
  - In previous example, $\#(s(x))$ is a *blocking structure* for voicing...
  - while sonorant(x) *licenses* voicing.

$$\begin{aligned} \text{voice}_o(x) \quad = \quad & \texttt{if } \text{sonorant}(x) \texttt{ then } \top \texttt{ else} \\ & \texttt{if } \#(s(x)) \texttt{ then } \bot \texttt{ else} \\ & \text{voice}(x) \end{aligned}$$

Advantages

- In contrast to OT, the evaluation of a BMRS hierarchy is necessarily *local* in nature.
  - Avoids the computational over-generation of global evaluation (Frank and Satta, 1998; Gerdemann and Hulden, 2012; Lamont, 2018).
- BMRSs are guaranteed to describe a strict subclass of the regular functions (Bhaskar et al., 2020).

Demonstration 1: Elsewhere condition effects

- Stressed vowels in binary feet are long when followed by /iV/ and short elsewhere (Chomsky and Halle, 1968; Myers, 1987; Halle, 1995; Baković, 2006)

(3)	a.	re(mē̱di)al	*re(mĕ̱di)al
	b.	co(lō̱ni)al	*co(lŏ̱ni)al
	c.	(rā̱di)al	*(ră̱di)al

(4)	a.	(nă̱tu)ral	(cf. nā̱ture)
	b.	di(vĭ̱ni)ty	(cf. divī̱ne)
	c.	(ră̱di)cal	

Demonstration 1: Elsewhere condition effects

$\textsc{head}(x)$

$$
\begin{aligned}
\underline{\textsc{NonHigh}}\mathsf{CiV}(x) = {} & \texttt{if } \mathsf{high}(x) \texttt{ then } \bot \texttt{ else} \\
& \quad \texttt{if } \mathsf{C}(s(x)) \texttt{ then} \\
& \qquad \texttt{if } \mathsf{i}(s(s(x))) \texttt{ then } \mathsf{V}(s(s(s(x)))) \\
& \quad \texttt{else } \bot
\end{aligned}
$$

$$
\begin{aligned}
& \textsc{head}\&\underline{\textsc{NonHigh}}\mathsf{CiV}(x) = \\
& \texttt{if } \textsc{head}(x) \texttt{ then } \underline{\textsc{NonHigh}}\mathsf{CiV}(x) \texttt{ else } \bot
\end{aligned}
$$

Demonstration 1: Elsewhere condition effects

$$\begin{aligned}\mathsf{long}_{\mathsf{o}}(x) \quad = \quad & \texttt{if}\ \textsc{head}\&\underline{\textsc{NonHigh}}\mathsf{CiV}(x)\ \texttt{then}\ \top\ \texttt{else}\\ & \texttt{if}\ \textsc{head}(x)\ \texttt{then}\ \bot\ \texttt{else}\\ & \mathsf{long}(x)\end{aligned}$$

Demonstration 1: Elsewhere condition effects

$$\mathrm{long}_o(x) = \texttt{if } \textsc{head}\&\underline{\textsc{NonHigh}}\mathrm{CiV}(x) \texttt{ then } \top \texttt{ else}$$
$$\texttt{if } \textsc{head}(x) \texttt{ then } \bot \texttt{ else}$$
$$\mathrm{long}(x)$$

- Note that $\textsc{head}\&\underline{\textsc{NonHigh}}\mathrm{CiV}(x)$ *implies* $\textsc{head}(x)$.
- If $\textsc{head}(x)$ appeared higher in the hierarchy than $\textsc{head}\&\underline{\textsc{NonHigh}}\mathrm{CiV}(x)$, the latter would *never* be evaluated (and would therefore be extraneous).

Strict Substructure Ordering Principle (SSOP)

(5) If structure j is a strict substructure of structure i (i.e., structure i implies structure j but not vice versa), then i must precede j in the hierarchy in order to be evaluated.

Strict Substructure Ordering Principle (SSOP)

(5) If structure *j* is a strict substructure of structure *i* (i.e., structure *i* implies structure *j* but not vice versa), then *i* must precede *j* in the hierarchy in order to be evaluated.

- Captures the disjunctive nature of shortening and lengthening.
- This principle is inherent to the logic of how BMRSs are evaluated and does not need to be stipulated.

Demonstration 2: the typology of *NC̥

(6) *NC̥
No nasal/voiceless obstruent sequences (Pater, 2004)

- Various repair strategies are accounted for in OT with permutations of faithfulness constraints and *NC̥.
- With BMRS, this variation is captured with the placement of blocking and licensing structures.

Demonstration 2: the typology of *NC̥

$$\underline{\mathrm{N}}\mathrm{\c{C}}(x) = \ \texttt{if}\ \mathsf{nasal}(x)\ \texttt{then}\ \textsc{VoicelessObs}(s(x))\ \texttt{else}\ \bot$$

$$\mathrm{N}\underline{\mathrm{\c{C}}}(x) = \ \texttt{if}\ \textsc{VoicelessObs}(x)\ \texttt{then}\ \mathsf{nasal}(p(x))\ \texttt{else}\ \bot$$

Demonstration 2: the typology of *NC̥

$$\textsc{Obstruent}(x) = \texttt{if}\ \mathsf{sonorant}(x)\ \texttt{then}\ \bot\ \texttt{else}\ \top$$

$$\textsc{VoicelessObs}(x) = \texttt{if}\ \mathsf{voice}(x)\ \texttt{then}\ \bot\ \texttt{else}\ \textsc{Obstruent}(x)$$

Demonstration 2: the typology of *NC̥

(7) Indonesian fusion (Onn, 1980)
/mənpilih/ ↦ [məmilih], ‘to choose, vote’

(8)
$$\texttt{out}(x) = \texttt{if } \underline{\mathrm{N\underset{\circ}{C}}}(x) \texttt{ then } \bot \texttt{ else } \top$$
$$\mathsf{labial}_{\circ}(x) = \texttt{if } \underline{\mathsf{Nasal}}\mathsf{Labial}(x) \texttt{ then } \top \texttt{ else } \mathsf{labial}(x)$$

$\underline{\mathrm{N\underset{\circ}{C}}}$ **blocks** out

Demonstration 2: the typology of *NC̥

(9) Puyo Pungo Quechua post-nasal voicing (Orr, 1962)
/kampa/ $\mapsto$ [kamba], 'yours'

(10) $\text{voice}_o(x) \;=\; \texttt{if } \underline{\text{NC̥}}(x) \texttt{ then } \top \texttt{ else } \text{voice}(x)$

$\underline{\text{NC̥}}$ **licenses** voice

Demonstration 2: the typology of *NC̥

(11) Nasalization in Konjo (Austronesian; Sulawesi; Friberg and Friberg, 1991)

/aŋpinahaŋ/	↦	[əmminahaŋ]	‘to follow’
/aŋkanre/	↦	[əŋŋanre]	‘to eat’

(12)

$$\text{nasal}_o(x) = \texttt{if } \underline{\text{NC̥}}(x) \texttt{ then } \top \texttt{ else } \text{nasal}(x)$$

$$\text{voice}_o(x) = \texttt{if } \text{nasal}_o(x) \texttt{ then } \top \texttt{ else } \text{voice}(x)$$

$\underline{\text{NC̥}}$ **licenses** nasal, and nasal **licenses** voice

Demonstration 2: the typology of *NC̥

(13) Mandar gemination (Austronesian; Sulawesi; Mills, 1975)
/mantunu/ ↦ [mattunu], ‘to burn’

(14)

$$\text{nasal}_o(x) \quad = \quad \texttt{if } \underline{\text{N}}\text{C̥}(x) \texttt{ then } \bot \texttt{ else } \text{nasal}(x)$$

<u>N</u>C̥ **blocks** nasal

Demonstration 2: the typology of *NC̥

N$\underline{\text{C̥}}$ **blocks** out	fusion
N$\underline{\text{C̥}}$ **licenses** [voice]	post-nasal voicing
N$\underline{\text{C̥}}$ **licenses** [nasal]	nasalization
$\underline{\text{N}}$C̥ **blocks** [nasal]	gemination

Demonstration 2: the typology of *NC̥

	N̲C̥		NC̲̥	
	licensing	blocking	licensing	blocking
out		Swahili		Indonesian
[nasal]$_\circ$		Mandar	Konjo	
[voice]$_\circ$		attested?	Quechua	

Meaningful structures

(15) Meaningful Structures in BMRSs

$$\texttt{if } X \texttt{ then } B \texttt{ else } Y$$

X is meaningful iff **neither** of the following holds.
B is $\top$ and X implies Y,
or B is $\bot$ and X implies $\neg Y$.

Meaningful structures

(16) Meaningful Structures in BMRSs
A formula of the form

$$\texttt{if } X \texttt{ then } B \texttt{ else } Y$$

is meaningful iff **neither** of the following holds.
B is $\top$ and X implies Y,
or B is $\bot$ and X implies $\neg Y$.

e.g., $\text{nasal}_o(x) = \texttt{if } \underline{\text{N}}\text{C̥}(x) \texttt{ then } \top \texttt{ else } \text{nasal}(x)$

- By definition (16), this formula is *meaningless*.

Open questions

- What kinds of operations are needed to combine multiple BMRSs into a complete phonological grammar?
  - Composition?
  - Priority union?
  - Something else?

Open questions

- How can recursion be used to capture long-distance processes?

(17) Kikongo (Ao, 1991; Odden, 1994)

/ku-kin-ila/	[ku-kin-ina]	‘to dance for’
/ku-dumuk-ila/	[ku-dumuk-ina]	‘to jump for’
/ku-dumuk-is-ila/	[ku-dumuk-is-ina]	‘to make jump for’

(18)
$$\text{follows-nas}(x) = \begin{array}{l} \texttt{if}\ \#(x)\ \texttt{then}\ \bot\ \texttt{else} \\ \texttt{if}\ \text{nas}(p(x))\ \texttt{then}\ \top\ \texttt{else} \\ \text{follows-nas}(p(x)) \end{array}$$

Open questions

- Consequences of using non-linear representations?
  - Recall $\textsc{Head}(x)$

Open questions

- Extension of prior finite-state-based learnability results for subregular functions.
  - How can we learn a BMRS template from positive data?

Conclusion

- BMRS capture phonological generalizations in a local way, maintaining the desirable computational restrictiveness of subregularity in a way that is more intuitive from a phonological perspective.
- Unlike prior finite-state characterizations of subregular functions, the restrictiveness of BMRSs doesn't depend on determinism.
  - Enables the use of feature-based representations.
  - Captures relationship among multiple generalizations more transparently.

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