Are Phonological Functions Total or Partial?

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Formalizing phonological processes

- Standard assumption in computational phonology: phonological processes are total functions (Johnson 1972, Kaplan & Kay 1994, Heinz 2011, Chandlee 2014, Bale & Reiss 2018).

  (1) Schwa-epenthesis in word-initial obstruent clusters
  {(spaɪ, səpaɪ), (staɪ, sətaɪ), (skaɪ, səkaɪ), (saI, saI), (wwww, wwww) ... }

- Domain is Σ^*
- Aligns with the Richness of the Base (Prince & Smolensky 1993).

Limiting the domain

- Morpheme Structure Constraints (Chomsky & Halle 1968)
- Domain is $D \subsetneq \Sigma^*$

{(spaɪ, səpaɪ), (staɪ, sətaɪ), (skaɪ, səkaɪ), (saI, saI), (wwww, wwww)
... }

Vacuous rule application

UR	/spaɪ/	/plen/
Epenthesis	səpaɪ	—
SR	[səpaɪ]	[plen]

UR	/spaɪ/	/plen/
Epenthesis	səpaɪ	plen
SR	[səpaɪ]	[plen]

{(spaɪ, səpaɪ), (staɪ, sətaɪ), (skaɪ, səkaɪ), (sal, sal), (wwww, wwww)
... }

Function composition

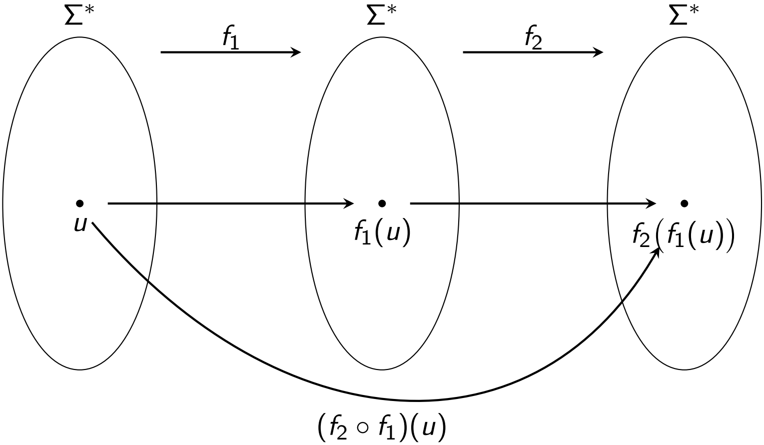
- Processes are combined via function composition (Johnson 1972, Kaplan & Kay 1994).

UR	$/u/$
f_1	x
f_2	y
f_3	z
SR	$[z]$

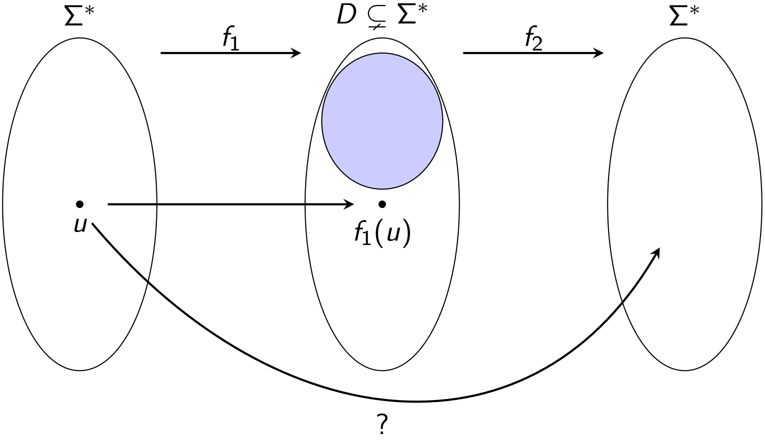
$$f_3(f_2(f_1(u))) = z$$

$$(f_3 \circ f_2 \circ f_1)(u) = z$$

Total functions simplify composition



Total functions simplify composition



Proposal

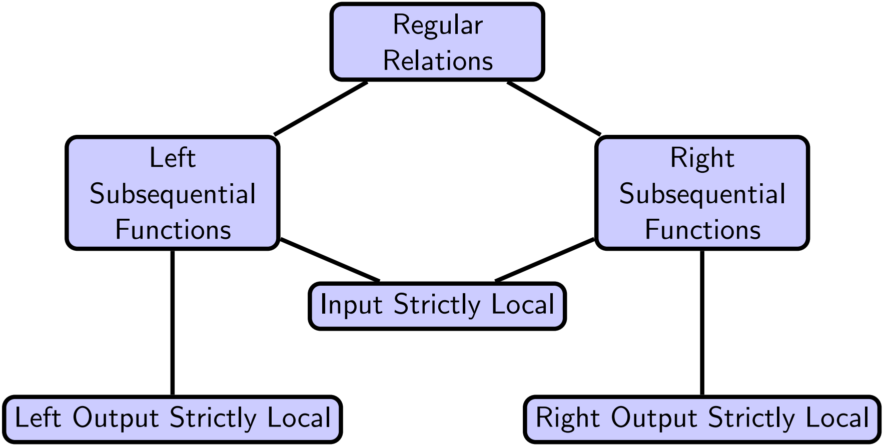
- In certain cases, a partial function analysis has advantages:
  - Better characterization of computational complexity
  - Learnability advantage
- These cases include 'non-local' phenomena with a bounded number of intervening segments.

(2) $a \rightarrow b \ / \ \_\_ \ X_0 \ d$

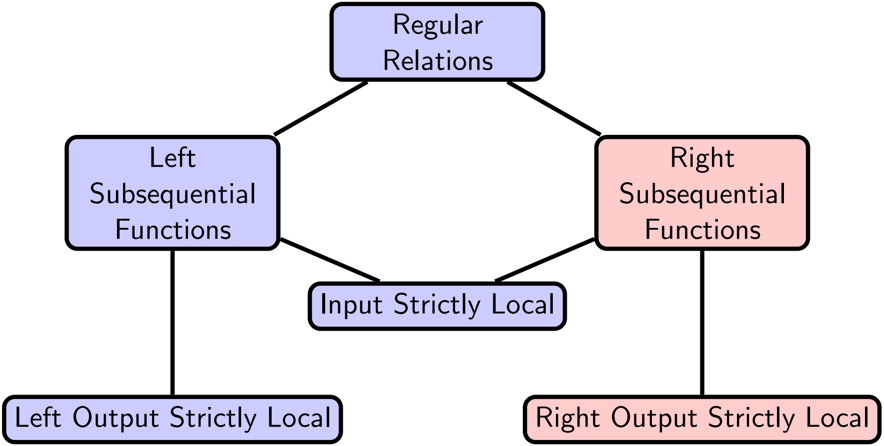
- As partial functions, such processes are computationally distinct from those described with rules like

(3) $a \rightarrow b \ / \ \_\_ \ \ldots \ d$

Hierarchy of functions



Hierarchy of functions



Right output strict locality

$a \to b \;/\; \_\_ \; b$	$a \to b \;/\; \_\_ \; a$
aaab $\mapsto$ bbbb	aaaa $\mapsto$ baba
# a a a b #	# a a a a #
b b b b	b a b a

Right subsequentiality

$$a \rightarrow b \ / \ \_\_ \ldots \ a \ \#$$

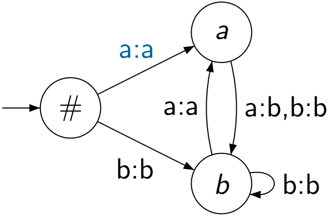
$$abba \mapsto bbba$$

#	a	b	b	a	#
	b	b	b	a	

FST characterizations

Right OSL

a → b / __ a

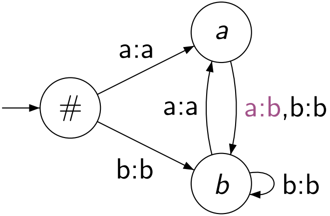


Input string:	#	a	a	a	a	#
Output string:					a	

FST characterizations

Right OSL

a → b / __ a

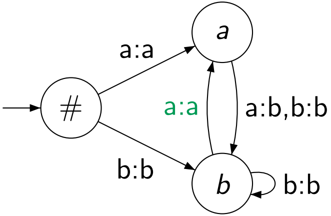


Input string:	#	a	a	a	a	#
Output string:				b	a	

FST characterizations

Right OSL

a → b / __ a

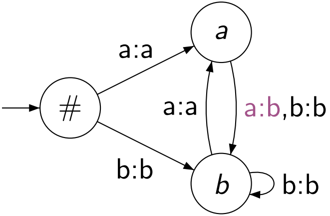


Input string:	#	a	a	a	a	#
Output string:			a	b	a	

FST characterizations

Right OSL

a → b / __ a

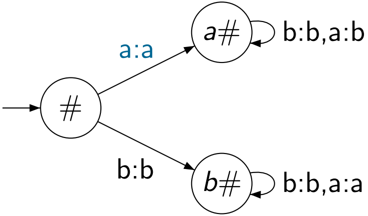


Input string:	#	a	a	a	a	#
Output string:		b	a	b	a	

FST characterizations

Right subsequential

a → b / ___ ... a#

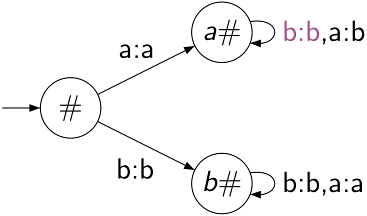


Input string:	#	a	b	b	a	#
Output string:					a	

FST characterizations

Right subsequential

a → b / __ … a#

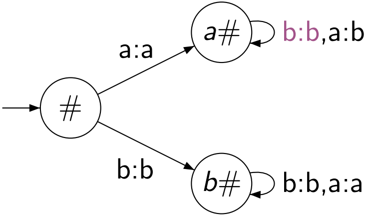


Input string:	#	a	b	b	a	#
Output string:				b	a	

FST characterizations

Right subsequential

a → b / ___ ... a#

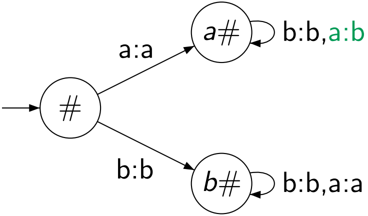


Input string:	#	a	b	b	a	#
Output string:			b	b	a	

FST characterizations

Right subsequential

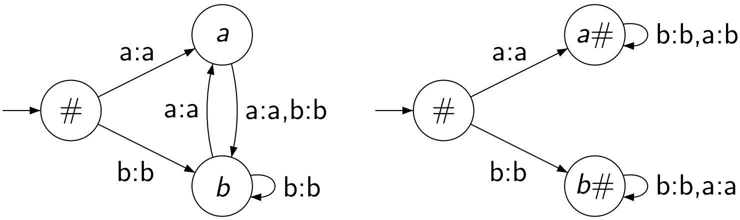
a → b / ___ ... a#



Input string:	#	a	b	b	a	#
Output string:		b	b	b	a	

FST characterizations

Right OSL　　　　Right subsequential



Isolating processes

- Computational classifications reflect the amount of information needed to model the input-output correspondence of a process.
- Total functions are meant to focus the classification on the process in question and nothing else.

#	a	w	a	n	#
	ã	w̃	ã	n	

#	a	n	n	n	#
	ã	n	n	n	

Vowel dissimilation

(4) Woleaian (Austronesian) (Sohn 1975)

a. /a/ $\rightarrow$ [e] / __ #

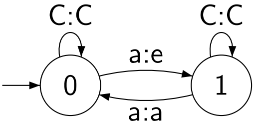
b. /a/ $\rightarrow$ [e] / __ C_0 a

(5)

a.	/matala/	[metale]	‘his eyes’
b.	/marama/	[merame]	‘moon’
c.	/yafaramami/	[yefaremami]	‘our shoulders’

Vowel dissimilation: total function

$\Sigma = \{C, a\}$
Right subsequential



#	m	a	t	a	l	a	#
	m	e	t	a	l	e	

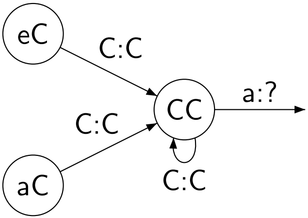
Vowel dissimilation: total function

OSL fails!

#	C	a	C	a	C	a	#
				a	C	e	

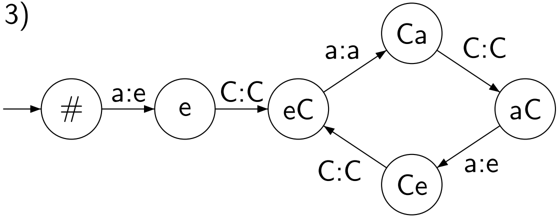
#	C	a	C	a	C	C	a	#
				?	C	C	e	

#	C	a	C	a	C	C	C	a	#
				?	C	C	C	e	



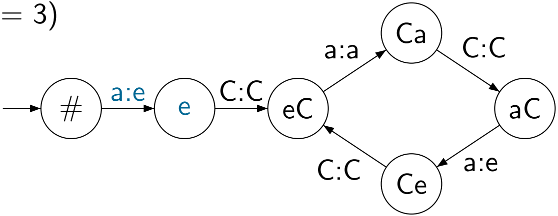
Vowel dissimilation: partial function

Domain: (Ca)*
Right OSL ($k = 3$)



Vowel dissimilation: partial function

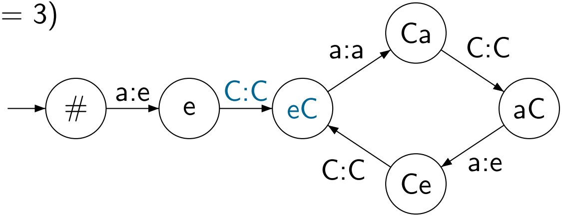
Domain: $(Ca)^*$
Right OSL ($k = 3$)



#	m	a	t	a	l	a	#
	m	e	t	a	l	e	

Vowel dissimilation: partial function

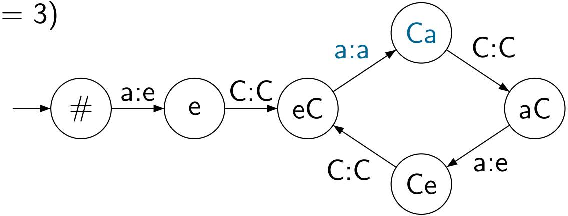
Domain: $(Ca)^*$
Right OSL ($k = 3$)



#	m	a	t	a	l	a	#
	m	e	t	a	l	e	

Vowel dissimilation: partial function

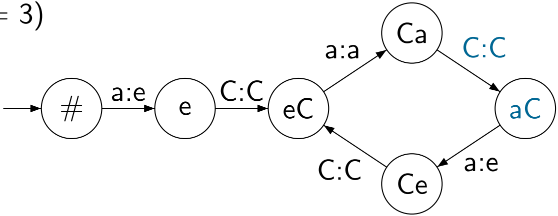
Domain: (Ca)*
Right OSL ($k = 3$)



#	m	a	t	a	l	a	#
	m	e	t	a	l	e	

Vowel dissimilation: partial function

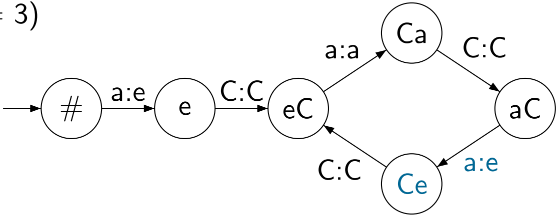
Domain: $(Ca)^*$
Right OSL $(k = 3)$



#	m	a	t	a	l	a	#
	m	e	t	a	l	e	

Vowel dissimilation: partial function

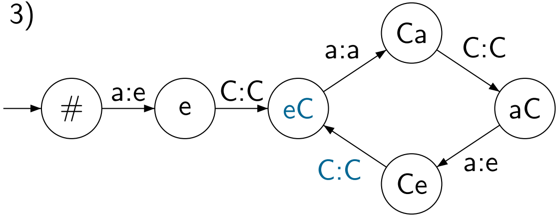
Domain: $(Ca)^*$
Right OSL ($k = 3$)



#	m	a	t	a	l	a	#
	m	e	t	a	l	e	

Vowel dissimilation: partial function

Domain: (Ca)*
Right OSL ($k = 3$)



#	m	a	t	a	l	a	#
	m	e	t	a	l	e	

Computational characterization

- This process is *output-oriented* and its trigger and target are included in a *bounded substring*.
- Classifying it as subsequential is only necessary to maintain the assumption of a total function.

Vowel harmony

(6) $V \rightarrow V_{\alpha F} \;/\; \_\_ \; C_0 \; V_{\alpha F}$

- Subsequential as a total function: one-sided context (Gainor et al. 2012, Heinz & Lai 2013)
    - Circumambient harmony is not subsequential (McCollum et al. (under review))
- As a partial function,
    - Without neutral vowels: OSL
    - With opaque vowels: OSL
    - With transparent vowels: Subsequential

Learning OSL functions

- Output Strictly Local Function Inference Algorithm (OSLFIA) (Chandlee et al. 2015)
- Proof of correctness assumes the target is a total function.

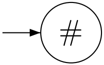
OSLFIA

$\Sigma = \{a,\ b\}$

$a \rightarrow b \ / \ b$ __

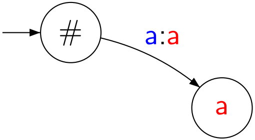
data:

{(a,a),(b,b),(aa,aa),(ab,ab),(ba,bb),(bb,bb),(aaa,aaa),(aab,aab),
(aba,abb),(abb,abb),(baa,bbb),(bab,bbb),(bba,bbb),(bbb,bbb)}



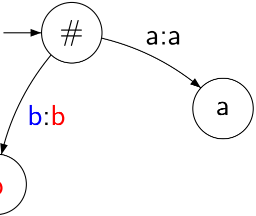
OSLFIA

$\Sigma = \{a, b\}$
$a \rightarrow b \;/\; b\;\_\_$
data:
{(a,a),(b,b),(aa,aa),(ab,ab),(ba,bb),(bb,bb),(aaa,aaa),(aab,aab),
(aba,abb),(abb,abb),(baa,bbb),(bab,bbb),(bba,bbb),(bbb,bbb)}



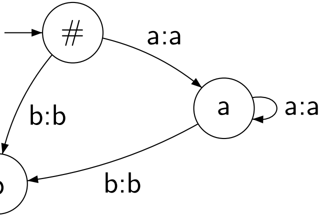
OSLFIA

$\Sigma = \{$a, b$\}$
a $\rightarrow$ b / b ___
data:
{(a,a),(b,b),(aa,aa),(ab,ab),(ba,bb),(bb,bb),(aaa,aaa),(aab,aab),
(aba,abb),(abb,abb),(baa,bbb),(bab,bbb),(bba,bbb),(bbb,bbb)}



OSLFIA

$\Sigma = \{a, b\}$
$a \rightarrow b \ / \ b \ \_\_$
data:
{(a,a),(b,b),(aa,aa),(ab,ab),(ba,bb),(bb,bb),(aaa,aaa),(aab,aab),
(aba,abb),(abb,abb),(baa,bbb),(bab,bbb),(bba,bbb),(bbb,bbb)}



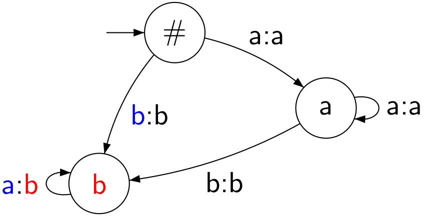
OSLFIA

$\Sigma = \{a, b\}$

a $\rightarrow$ b / b __

data:

{(a,a),(b,b),(aa,aa),(ab,ab),(ba,bb),(bb,bb),(aaa,aaa),(aab,aab),
(aba,abb),(abb,abb),(baa,bbb),(bab,bbb),(bba,bbb),(bbb,bbb)}

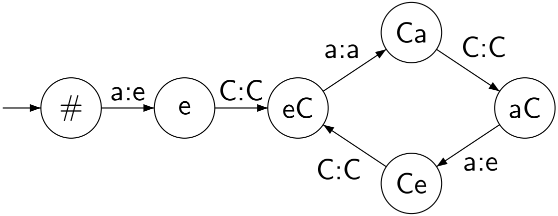


Data required to learn total functions

- For a given Σ and k, the sample must include input strings for all possible sequences from Σ up to length $k+2$
- For example, if $\Sigma = \{C, a\}$ and $k = 3$, needs to see inputs like CaCa, but also CCCCC, aaaC, etc.

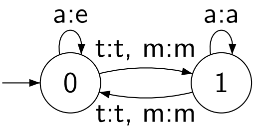
Learning partial functions

- Given strings of up to three syllables from $(Ca)^*$, with $C = \{t, m\}$, OSLFIA returns a partial function FST that is equivalent to the target:



Learning partial functions

- Onward Subsequential Transducer Inference Algorithm (OSTIA) (Oncina et al. 1993) learns total subsequential functions.
- OSTIA-D (Oncina & Varó 1996)) learns partial functions given an FSA representation of the domain.
- Given the same data sample as OSLFIA, *both* OSTIA and OSTIA-D return:



Learning partial functions

- When targeted as an OSL function, vowel dissimilation is learnable with less and more plausible data.
- Targeting it as an OSL function is only possible if it's a partial function.
- For future work:
  - Formally, what types of partial functions are learnable by OSLFIA?
  - Why doesn't domain information help OSTIA in these cases?

When to use a partial function?

- What are the formal properties of maps for which a partial function analysis should be considered?
  - Always iterative/output-oriented?
  - What is the nature of the limit on the domain?
- What about composition?

Locality and representation

- Additional function classes have been proposed for non-local processes, including A-ISL (Chandlee & Jardine 2019) and O-TSL (Burness & McMullin 2019).
- These classes are subsets of subsequential, but more powerful than OSL.
- Partial functions are a way to preserve locality without adding complexity.

Acknowledgements

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