Locality in Phonological Interactions: A Computational Account

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January 18, 2017

Introduction

- Many phonological input-output maps can be characterized by a small, computationally-restrictive and learnable class of functions, including **opaque interactions**.
- This result is part of a larger research program that aims to understand the computational nature of phonological patterns and how their computational properties can inform how they are learned.

Outline

- Establish a framework for classifying phonological maps based on their computational complexity.
  - Subregular hierarchy of function classes: ISL, OSL, TSL
- Use this framework to present a computational analysis of opaque map interactions.
  - 7 distinct cases of opacity are shown to be ISL functions.

Computational complexity of phonological patterns

- What do we know about the computational complexity of phonotactics and phonological maps?
- Starting point: both are *regular* (i.e., finite state describable)
- Proposal: both are in fact *subregular* (i.e., describable with proper subsets of regular)
- Why is this restriction desirable?
  - Better fit to the typology
  - Learnability advantages

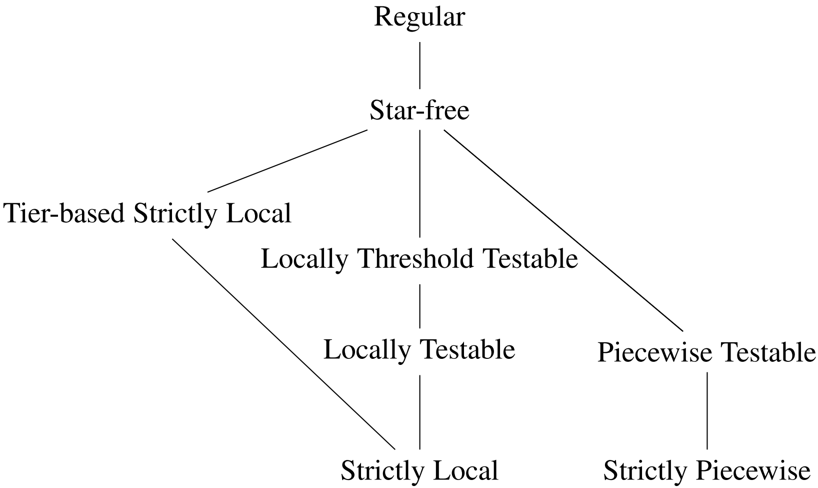
Formal languages and phonotactics

- A formal language is a set of strings.
- A phonotactic constraint can be modeled with the set of strings that do not violate it.

(1) { amba, nanta, ama, ... }

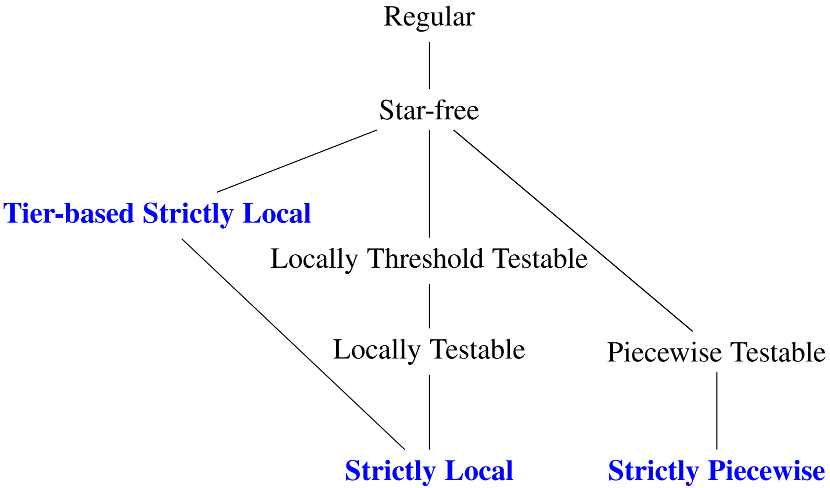
- What is the *complexity* of the formal languages that model phonotactics?

Subregular hierarchy of languages



(Rogers and Pullum, 2011; Rogers et al., 2013; Heinz, 2010; Heinz et al., 2011; McMullin, 2016)

Subregular hierarchy of languages



(Rogers and Pullum, 2011; Rogers et al., 2013; Heinz, 2010; Heinz et al., 2011; McMullin, 2016)

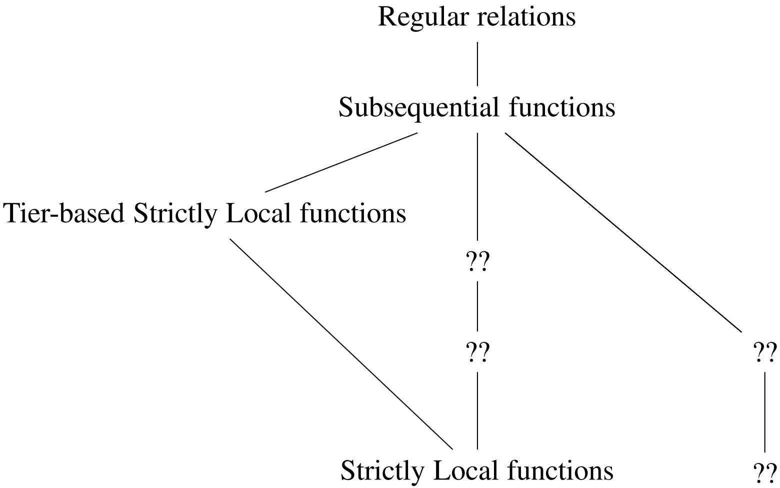
Phonological maps

- A relation/function/map is a set of string *pairs*.
- A phonological map (i.e., ‘process’) is a (UR, SR) pair.

(2) { (anba, amba), (nanta, nanta), (ama, ama), ... }

- What is the *complexity* of phonological maps?

Subregular hierarchy of maps



(Johnson, 1972; Kaplan and Kay, 1994; Mohri, 1997; Chandlee, 2014; Chandlee et al., 2017)

Computational complexity of phonological maps

REGULAR RELATIONS (Johnson, 1972; Kaplan and Kay, 1994)

↓

SUBSEQUENTIAL FUNCTIONS (Mohri, 1997)

↓

TIER-BASED STRICTLY LOCAL FUNCTIONS (Chandlee et al., 2017)

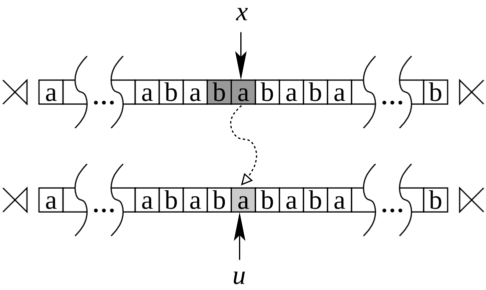
↓

STRICTLY LOCAL FUNCTIONS (Chandlee, 2014)

Strictly Local functions

- Output string is computed *locally*, depending only on a bounded number of previous segments.
- Two varieties:
  - Input Strictly Local: output depends on previous input symbols
  - Output Strictly Local: output depends on previous output symbols

Input Strictly Local (ISL) function



Example: Input Strictly Local (ISL) function

(3) Korean (Lee and Pater, 2008)
/pa**pm**ul/ $\mapsto$ [pa**mm**ul] ‘rice water’

$\rtimes$	p	a	p	m	u	l	$\ltimes$
	λ						

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Example: Input Strictly Local (ISL) function

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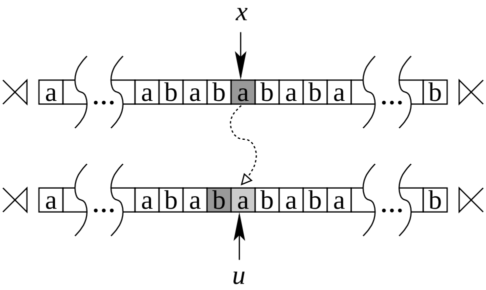
Window size is 2: this map is 2-ISL.

Output Strictly Local function



Example: Output Strictly Local (OSL) function

(4) Johore Malay (Onn, 1980)
/pəŋawasan/ ↦ [pəŋãw̃ãsan] ‘supervision’

⋊	p	e	ŋ	a	w	a	s	a	n	⋉
	p									

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	p	e	ŋ	ã	w̃					

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Window size is 2: this map is 2-OSL.

Long-distance (unbounded) assimilation

(5) Kikongo (Meinof, 1932; Odden, 1994; Rose and Walker, 2004)

a. /tu**n**iki**d**i/ $\mapsto$ [tu**n**iki**n**i] ‘we ground’

b. /kudu**m**ukisi**l**a/ $\mapsto$ [kudu**m**ukisi**n**a] ‘to cause to jump for’

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⋊	t	u	n	i	k	i	d	i	⋉
	t	u	n	i	k	i	n		

⋊	k	u	d	u	m	u	k	i	s	i	l	a	⋉
	k	u	d	u	m	u	k	i	s	i	l		

Example: Tier-based Strictly Local (TSL) function

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- Designate a subset of the alphabet as the *tier*: T = {n, m, d, l}

$\rtimes$	t	u	n	i	k	i	d	i	$\ltimes$
	t								

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	t	u	n	i					

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⋊	t	u	n	i	k	i	d	i	⋉
	t	u	n	i	k	i	n		

Long-distance assimilation with blocking

Slovenian (Jurgec, 2011; McMullin, 2016)

a. **s**pi**ʃ** ↦ **ʃ**pi**ʃ** ‘(you) sleep’

b. **z**aklɔni**ʃtʃ**e ↦ **ʒ**aklɔni**ʃtʃ**e ‘bomb shelter’

c. na**s**i**t**i**ʃ** ↦ na**s**i**t**i**ʃ** ‘(you) feed’

- The blocking segments go on the tier.

⋊	n	a	s	i	t	i	ʃ	⋉
							ʃ	

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⋊	n	a	s	i	t	i	ʃ	⋉
			s	i	t	i	ʃ	

Example classifications

- ISL and OSL are argued to include all local phonological processes (assimilation, dissimilation, insertion, deletion, metathesis) (Chandlee, 2014; Chandlee et al., 2015)
- Many morphological ‘maps’ are also ISL (Chandlee, under review)
- TSL is conjectured to include long-distance processes

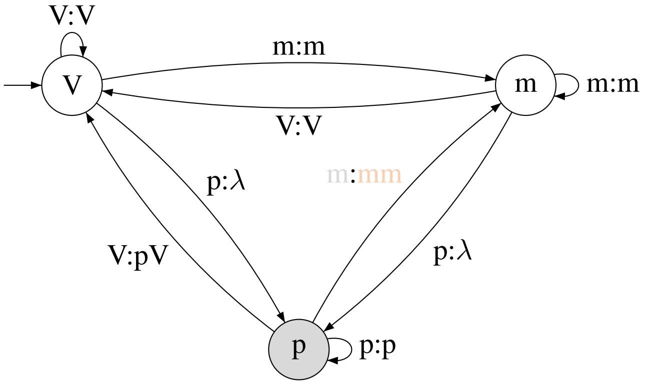
Characterizations

Class	Finite-state	Language-theoretic	Logical
ISL	✓	✓	✓
OSL	✓	✓	
TSL	✓	✓	

(Chandlee (2014); Chandlee et al. (2014, 2015, 2017), Chandlee and Lindell (to appear))

Input Strictly Local (ISL): FST characterization

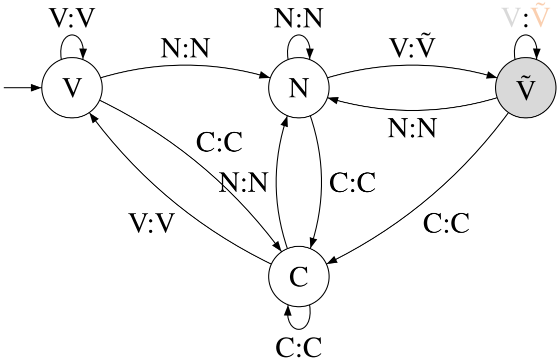
(7) /pa**pm**ul/ $\mapsto$ [pa**mm**ul]



⋊	p	a	p	m	u	l	⋉
	λ	pa	λ	mm			

Output Strictly Local (OSL): FST characterization

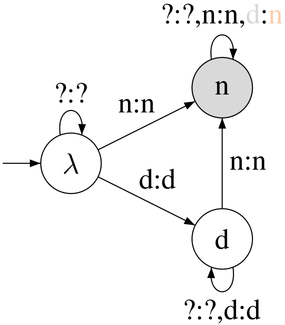
(8) /pəŋawasan/ ↦ [pəŋãw̃ãsan]



⋊	p	e	ŋ	a	w	a	s	a	n	⋉
	p	e	ŋ	ã	w̃					

Tier-based Strictly Local (TSL): FST characterization

(9) /tu**n**iki**d**i/ $\mapsto$ [tu**n**iki**n**i]



⋊	t	u	n	i	k	i	d	i	⋉
	t	u	n	i	k	i	n		

FST Characterizations

REGULAR RELATIONS (describable with FSTs)

↓

SUBSEQUENTIAL FUNCTIONS (describable with deterministic FSTs)

↓

TIER-BASED STRICTLY LOCAL FUNCTIONS (states are $T^{\leq k-1}$)

↓

STRICTLY LOCAL FUNCTIONS (transitions follow recent input/output)

Map interactions

What about map interactions?

Case study: opaque phonological generalizations

- Opaque phonological generalizations result from the interaction of two processes, one of which is not ‘surface-true’.
- The 7 distinct types of opacity from McCarthy (2007); Baković (2007); Kavitskaya and Staroverov (2010) can all be described with ISL maps.
- The needed information for determining the correct output at any point in the computation is found in a contiguous window of the input.

Case study: opaque phonological generalizations

(10) Counterbleeding

a. Lowering: [+long] $\rightarrow$ [$-$high]

b. Shortening: V $\rightarrow$ [$-$long] / __ C #

(11) Yokuts (McCarthy, 1999)
/ʔili:l/ $\mapsto$ [ʔilel], 'might fan'

- Lowering over-applies.

Case study: opaque phonological generalizations

(12) Counterbleeding

a. Lowering: [+long] $\rightarrow$ [$-$high]

b. Shortening: V $\rightarrow$ [$-$long] / ___ C #

(13) Yokuts (McCarthy, 1999)

/ʔili:l/ $\mapsto$ [ʔilel], ‘might fan’

⋊	ʔ	i	l	i:	l	⋉
	ʔ					

Case study: opaque phonological generalizations

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$\rtimes$	ʔ	i	l	i:	l	$\ltimes$
	ʔ	i	l	λ		

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	ʔ	i	l	λ	λ	el

Case study: opaque phonological generalizations

(14) Counterfeeding on focus

a. Deletion: $i \rightarrow \emptyset$ / ___ CV
b. Raising: $a \rightarrow i$ / ___ CV

(15) Bedouin Arabic (McCarthy, 1999)
/katab/ $\mapsto$ [kitab], ‘he wrote’

- Deletion underapplies.

Case study: opaque phonological generalizations

(16) Counterfeeding on focus

a. Deletion: i $\rightarrow \emptyset$ / __ CV

b. Raising: a $\rightarrow$ i / __ CV

(17) Bedouin Arabic (McCarthy, 1999)
/katab/ $\mapsto$ [kitab], 'he wrote'

⋊	k	a	t	a	b	⋉
	k					

Case study: opaque phonological generalizations

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	k	λ				

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Case study: opaque phonological generalizations

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$\rtimes$	k	a	t	a	b	$\ltimes$
	k	λ	λ	it	λ	ab

Case study: opaque phonological generalizations

(18) Self-destructive feeding (Baković, 2007)

a. Epenthesis: $\emptyset \rightarrow$ i / C(+)__C#

b. Deletion: k $\rightarrow \emptyset$ / V__+V

(19) Turkish (Sprouse, 1997)
/bebek+n/ $\mapsto$ [bebein], 'your baby'

- Deletion destroys the environment for epenthesis.

Case study: opaque phonological generalizations

(20) Self-destructive feeding (Baković, 2007)

a. Epenthesis: $\emptyset \rightarrow$ i / C(+)__C#

b. Deletion: k $\rightarrow \emptyset$ / V__+V

(21) Turkish (Sprouse, 1997)
/bebek+n/ $\mapsto$ [bebein], ‘your baby’

$\rtimes$	b	e	b	e	k	+	n	$\ltimes$
	b							

Case study: opaque phonological generalizations

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a. Epenthesis: $\emptyset \rightarrow$ i / C(+)__C#

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Case study: opaque phonological generalizations

(20) Self-destructive feeding (Baković, 2007)

a. Epenthesis: $\emptyset \rightarrow$ i / C(+)__C#

b. Deletion: k $\rightarrow \emptyset$ / V__+V

(21) Turkish (Sprouse, 1997)
/bebek+n/ $\mapsto$ [bebein], 'your baby'

⋊	b	e	b	e	k	+	n	⋉
	b	e	b					

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$\rtimes$	b	e	b	e	k	+	n	$\ltimes$
	b	e	b	e	λ			

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	b	e	b	e	λ	λ	λ	in

Summary: ISL Opaque Maps

Opacity Type	**Language**	**k-value**
cross-derivational feeding	Lithuanian	$k = 2$
counterbleeding	Yokuts	$k = 3$
fed counterfeeding	Tundra Nenets	$k = 3$
counterfeeding on environment	Bedouin Arabic	$k = 3$
counterfeeding on focus	Bedouin Arabic	$k = 3$
non-gratuitous feeding	Classical Arabic	$k = 4$
self-destructive feeding	Turkish	$k = 5$

Implications for phonological learning

- The regular relations are not learnable from positive data...
- but the ISL functions are (Chandlee et al., 2014; Jardine et al., 2014)

Conclusion

- There is an increasing amount of evidence that phonological maps are subregular in nature, and this property is not dependent on analyses of individual generalizations.
- In particular, opaque map interactions belong to one computationally restrictive and learnable subregular class, the ISL functions.

Future work

- Complete the hierarchy of subregular maps, particularly the regions for long-distance phenomena
- Further explore the logical characterizations of these classes
- Investigate other types of map interaction
  - Blocking
  - Co-existing long-distance maps with and without distinct tiers

Acknowledgements

- Jeffrey Heinz (University of Delaware)
- Adam Jardine (Rutgers University)
- Jim Rogers (Earlham College)
- Rémi Eyraud (Laboratoire D'Informatique Fondamentale de Marseille)
- Kevin McMullin (University of Ottawa)
- Steven Lindell (Haverford College)

Appendix: Additional examples of opacity

(22) Counterfeeding on environment

a. Raising: a $\rightarrow$ i / __ CV

b. Vocalization: G $\rightarrow$ V / C __ #

(23) Bedouin Arabic (McCarthy, 1999)
/badw/ $\mapsto$ [badu], 'Bedouin'

- Raising underapplies.

Appendix: Additional examples of opacity

(24) Counterfeeding on environment

a. Raising: $a \rightarrow i$ / ___ CV

b. Vocalization: $G \rightarrow V$ / C ___ #

(25) Bedouin Arabic (McCarthy, 1999)
/badw/ $\mapsto$ [badu], 'Bedouin'

⋊	b	a	d	w	⋉
	b				

Appendix: Additional examples of opacity

(24) Counterfeeding on environment

a. Raising: a $\rightarrow$ i / ___ CV

b. Vocalization: G $\rightarrow$ V / C ___ #

(25) Bedouin Arabic (McCarthy, 1999)
/badw/ $\mapsto$ [badu], ‘Bedouin’

$\rtimes$	b	a	d	w	$\ltimes$
	b	λ			

Appendix: Additional examples of opacity

(24) Counterfeeding on environment

a. Raising: a $\rightarrow$ i / ___ CV

b. Vocalization: G $\rightarrow$ V / C ___ #

(25) Bedouin Arabic (McCarthy, 1999)
/badw/ $\mapsto$ [badu], ‘Bedouin’

$\rtimes$	b	a	d	w	$\ltimes$
	b	λ	λ		

Appendix: Additional examples of opacity

(24) Counterfeeding on environment

a. Raising: a $\rightarrow$ i / ___ CV

b. Vocalization: G $\rightarrow$ V / C ___ #

(25) Bedouin Arabic (McCarthy, 1999)
/badw/ $\mapsto$ [badu], ‘Bedouin’

$\rtimes$	b	a	d	w	$\ltimes$
	b	λ	λ	ad	

Appendix: Additional examples of opacity

(24) Counterfeeding on environment

a. Raising: a $\rightarrow$ i / __ CV

b. Vocalization: G $\rightarrow$ V / C __ #

(25) Bedouin Arabic (McCarthy, 1999)
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$\rtimes$	b	a	d	w	$\ltimes$
	b	λ	λ	ad	u

Appendix: Additional examples of opacity

(26) Non-gratuitous feeding (Baković, 2007)

a. Vowel epenthesis: $\emptyset \rightarrow V_i$ / # __ CCV_i

b. Glottal epenthesis: $\emptyset \rightarrow$ ʔ / #__V

(27) Classical Arabic (McCarthy, 2007)
/ktub/ $\mapsto$ [ʔuktub], ‘write.MascSg!’

$\rtimes$	k	t	u	b	$\ltimes$
	λ				

Appendix: Additional examples of opacity

(26) Non-gratuitous feeding (Baković, 2007)

a. Vowel epenthesis: $\emptyset \rightarrow V_i$ / # ___ CCV_i

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⋊	k	t	u	b	⋉
	λ	λ			

Appendix: Additional examples of opacity

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a. Vowel epenthesis: $\emptyset \rightarrow V_i$ / # ___ CCV_i

b. Glottal epenthesis: $\emptyset \rightarrow$ ʔ / #___V

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/ktub/ $\mapsto$ [ʔuktub], ‘write.MascSg!’

⋊	k	t	u	b	⋉
	λ	λ	ʔuktu		

Appendix: Additional examples of opacity

(28) Cross-derivational feeding (Baković, 2007)

a. Epenthesis: $\emptyset \rightarrow$ i / K_i __ K_i

b. Assimilation: K $\rightarrow$ [+voice] / __ D

(29) Lithuania (Odden, 2005)

a. /at-taiki:ti/$\rightarrow$[atitaiki:ti], 'to make fit well'

b. /ap-gauti/ $\mapsto$ [abgauti], 'to deceive'

c. /ap-berti/ $\mapsto$ [apiberti], 'to strew all over'

$\rtimes$	a	p	b	e	r	t	i	$\ltimes$
	a	λ	pib					

Appendix: Additional examples of opacity

(30) Fed Counterfeeding (Kavitskaya and Staroverov, 2010)

a. Glottalization: $\{$t, d, s, n, ŋ$\} \rightarrow$ ʔ / __ #

b. Deletion: ʌ $\rightarrow \emptyset$ / __ (ʔ) #

(31) Tundra Nenets

a. /tasʌ/ $\mapsto$ [tas], ‘whole’

b. /tʲimjʌs/ $\mapsto$ [tʲimjʔ], ‘it rotted’

$\rtimes$	t	a	s	ʌ	$\ltimes$
	t	a	λ	λ	s

$\rtimes$	tʲ	i	m	j	ʌ	s	$\ltimes$
	tʲ	i	m	j	λ	λ	ʔ

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