

Central Contribution

Phonological maps from underlying to surface forms can be represented using logical formulae over graph transductions. Doing so provides a very natural extension from local to long-distance phonology, allowing us to build on what is known about the computational nature of local phonology (Heinz, 2007, 2009; Chandlee, 2014) to establish a more complete understanding of what is phonologically possible.

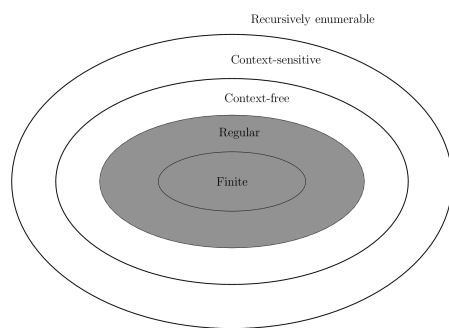
Computational Approach

- Both rule- and constraint-based theories of generative phonology concur on the existence of a map from input (underlying) to output (surface) forms.
- We model these maps with *functions* with the goal of identifying computational properties that are independent of any grammatical formalism (rules or constraints).

(1) Let f be a function which voices obstruents after nasals.
Example (Quechua, Pater (2004)) $f(\text{kampa}) = [\text{kamba}]$, 'yours'

Q: What class of functions does f belong to?

- Identifying the most *restrictive* set of functions needed for phonological maps leads to a better characterization of the components of phonological grammars.
- Previous work showed that phonological maps are regular (Johnson, 1972; Kaplan and Kay, 1994), with substantial evidence indicating they are in fact subregular (Mohri (1997); Heinz and Lai (2013); Chandlee (2014); Jardine (to appear)).

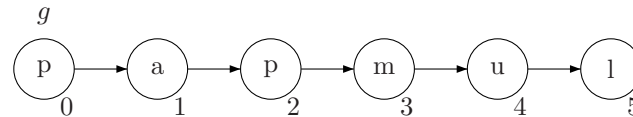


(Chomsky, 1956)

- A variety of formalisms can be used to define the exact same class of functions (i.e., finite-state automatic, logical, algebraic, formal language-theoretic).

Local/Bounded Maps

(2) Korean (Lee and Pater 2008)
 $/\text{pappmul}/ \mapsto [\text{pammul}]$ 'rice water'

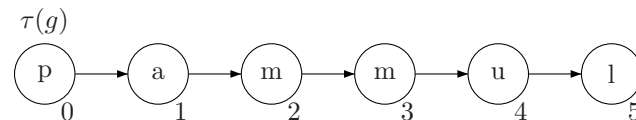


- Set of positions: $\{0, 1, 2, 3, 4, 5\}$
- Successor relation: $\{(0,1), (1,2), (2,3), (3,4), (4,5)\}$
- Labeling function/segment predicates: $p(x) = \{0, 2\}$, $a(x) = \{1\}$, $m(x) = \{3\}$, $u(x) = \{4\}$, $l(x) = \{5\}$
- First define natural class predicates (e.g., $N(x) \equiv n(x) \vee m(x) \vee \eta(x) \dots$), and then define 'substring predicates':
 $CN(x) \equiv C(x) \wedge (\exists y)[N(y) \wedge x \triangleleft y]$
- Formulae define the output graph in terms of the input graph (Engelfriet and Hooeboom, 2001):

$$\varphi_N^0(x) \equiv N(x) \vee CN(x)$$

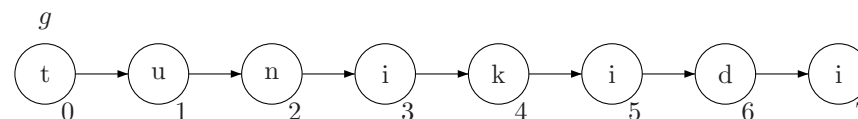
$$\varphi_C^0(x) \equiv C(x) \wedge \neg CN(x)$$

$$\varphi_V^0(x) \equiv V(x)$$



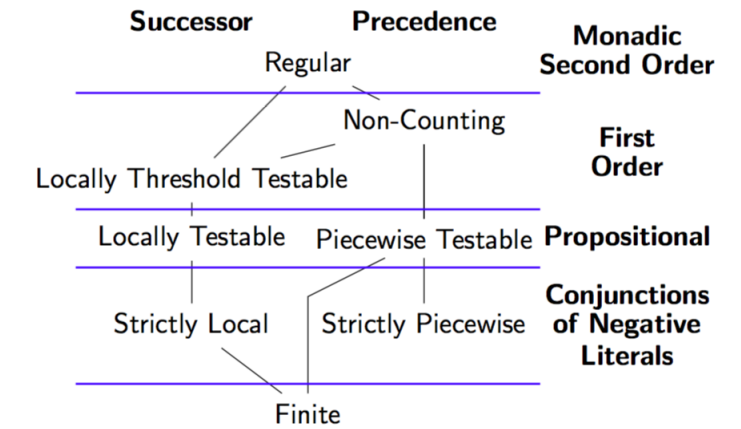
Long-distance/Unbounded Maps

(3) Kikongo (Meinhof 1932, Odden 1994, Rose and Walker 2004)
 $/\text{tunikidi}/ \mapsto [\text{tunikini}]$ 'we ground'



- Following Heinz (2010), the distinction between local and long-distance phonology is a shift from successor to precedence.
- Precedence relation: $\{(0,1), (0,2) \dots (1,3) \dots (2,6)\}$
- Subsequence predicate: $ND(x) \equiv D(x) \wedge (\exists y)[N(y) \wedge y < x]$
- $\varphi_N^0(x) \equiv N(x) \vee ND(x)$

Subregular Hierarchy



(Rogers & Pullum 2011, Rogers et al. 2013)

Conclusions and Open Questions

- Logical characterizations of phonological maps indicate that the difference between local and long-distance is one of representation, not computational complexity.
- Previous findings for phonotactics (Heinz, 2009, 2010) suggest that phonology is restricted to the lowest regions of the subregular hierarchy - what are the equivalent regions for subregular phonological maps?
- Learnability results exist for local maps (Chandlee et al., 2014; Jardine et al., 2014) based on an FST characterization; a corresponding FST for LD maps will lead to a more complete computational learning model.

Selected References

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